MECH 880: Numerical Methods

Professor: Dr. Florin Bobaru

Author: Ricardo Jacome

**Wavelet Transforms – Introduction and Application in Signal Analysis**

**Abstract:** Signal analysis is used to decompose a signal into elements such as noise or other elementary functions in order to filter or categorized the sampled information. Many methods for decomposition and filtering exists such as Fourier Series and Fast Fourier Transforms (FFT). This paper takes a more general method known as Wavelets with its corresponding Wavelet Transforms. A basic review of the method is provided along with some examples and applications of Wavelets in engineering applications such as acceleration analysis. The method is explored with the MATLAB wavelet tool that permits decomposition and analysis of the accelerations experimented by a vehicle under harsh braking scenarios such as different surface roads.

**Background**

Waves are an oscillating function defined in time and space, such as sinusoids. These sinusoids are used as basis functions to construct any periodic signal. Such construction is known as a Fourier Series representation. This is done to filter signals by finding the frequency content that represents the desired signal and removing all other frequencies that are categorized as noise. This method has limitations in terms of locating the time event of the frequencies captured. Similarly, this method is limited to periodic functions only. For this reason, wavelets were introduced to compensate for the limitations on representing signals with Fourier Series. Wavelets can be interpreted as a small wave with its energy concentrated in a position in time. These wavelets serve as the new basis functions that can decompose signals that are non-periodic while maintaining information about both frequency and time contents.

To exemplify a signal decomposition in wavelets, a Fourier series decomposition is shown in Figure for a direct comparison. Instead of sines and cosines, the wavelet decomposition is composed of two functions: The Scaling Function and the Wavelet Function . Similar to sines and cosines, both the scaling and wavelet functions are orthogonal functions that are linearly independent of each other.

The coefficients c and d can be found through the principle of inner product for orthogonal functions. These coefficients receive the name of Discrete Wavelet Transform (DWT) Coefficients, which is analogous to the FFT Coefficients for signal decomposition. Instead of a basis function in the form of a sine or a cosine, wavelets have the advantage of an infinite range of Wavelet functions available for signal construction. In general, the scaling function has the following form below, where consists of scaling coefficients.

To define a Wavelet Function, the scaling function from before is used with a shift, along with some Wavelet Coefficients as shown below.

Depending on the signal to be analyzed, there are many wavelet functions that can be used. Similarly, there are families of wavelets within each type. These families each are classified by how coarse the approximation is. The most popular, available in MATLAB library are:

|  |  |
| --- | --- |
| Wavelet Family Name | |
| Haar Wavelet | Biorthogonal Wavelets |
| Daubechies Wavelets | Reverse Biorthogonal Wavelets |
| Symlets | Meyer Wavelet |
| Coiflets | Gaussian Wavelets |
| Mexican Hat Wavelet | Morlet Wavelet |
| Complex Gaussian Wavelet | Shannon Wavelets |
| Frequency B-Spline Wavelets | Complex Morlet Wavelets |

To make a comparison in between the analytical formulation and the embedded function, the Morlet and Mexican Hat Wavelets are shown below.



**Applications**

Like Fourier Series, Wavelets are used to decompose signals into components for applications such as filtering, machine vision, fingerprint compression, musical tones, and image recognition. For this report, the application of filtering was selected for acceleration analysis.

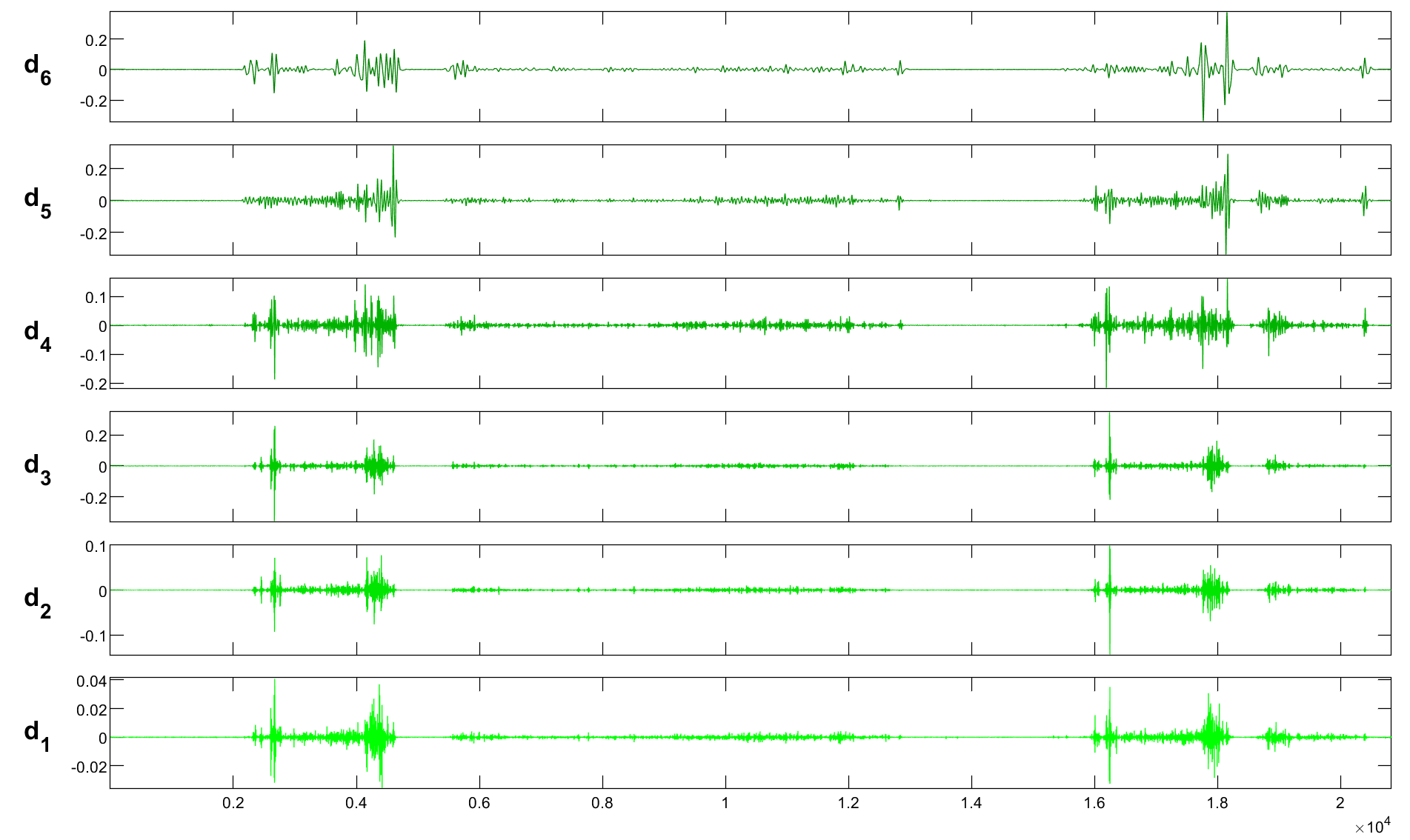
The purpose of the analysis is to explore the acceleration behavior of a vehicle under braking scenarios while traversing split surfaces of contact. The coefficient of friction for the road has a crucial influence in the level of braking a car can perform. Thus, split surface scenarios contribute to different levels on braking on each wheel individually. These scenarios motivated an investigation into the acceleration profiles a vehicle demonstrates under split surfaces.

The acceleration data comes from a testing scenario performed at Mwrsf Testing Grounds. An accelerometer was placed near the center of gravity of the vehicle, and the testing scenario consisted of a vehicle driving up to 45 mph, entering a split surface in which both right wheels of the vehicle are in gravel, and both left wheels are in concrete.

Ideal acceleration profiles are in the form of linear functions, which are impossible to obtain through testing due to the noisy nature of the instrumentation. For this reason, techniques such as Fast Fourier Transforms are often limited in how to categorize portions of the signal. An FFT is shown below to compare filtered signals from an FFT and a Wavelet.



As it is noticed, the reconstructed signal by the FFT method, offers an oscillatory behavior that captures the “overall shape” of the curve, but does not capture the linear behavior of the actual signal as an ideal acceleration profile would offer. For the Wavelet comparison, the acceleration data was analyzed with the Wavelet Analyzer from MATLAB. The Wavelet Family selected was Coiflet 2, with up to 6 levels of decomposition, with a sample Coiflet 2 function shown below. The choice of this wavelet was due to its “linear” shape that can capture linear slopes better than other Wavelets.





The report has to have the following sections:

Abstract, a clear introduction and description of methods, examples and applications, brief conclusions, and references. No more than 7pages are allowed. An extra 3 pages are allowed for including the code (can be double column if needed). Two choices are possible for the applications and example

Implement the method(s) yourself; test on examples; compare with other methods and/or exact solutions (using the programming language of your choice)and comment of advantages/disadvantages compared with other methods.

Use software packages (MATLAB, Mathematica, Maple, etc.) To build more complex applications that provide solutions for a specific applied problem of your choice.